White Paper No. 12

Fan Laws and Fan Correlations

Question: Is there a system in fan catalogues?

Introduction

Fans are common helpers for cooling electronic devices. But it's like a zoo. There are big and small, strong and weak, fast and slow. The fan laws (or affinity curves) combine, among other things, diameter *D*, volume flow G, static pressure P and speed *N* of propeller fans. A search on the Internet provides you with a large number of hits (e.g. [1]). But they keep repeating that the "laws" compare the fans relative to each other, only. The laws allow you to calculate a new data set #2 based on a reference data set #1 with known values of the variables. But, they are useless if you do not have set #1 data. I wondered whether and how these formulas fit in with the values we find in fan catalogues, and whether they can be quantitatively evaluated. I searched the Internet for more information, but found no paper. The goal of this article is to supplement the fan laws with numbers and to enable a quick evaluation, or prediction, in a spreadsheet. I call them “*fan correlations*”.

This is the subset of “laws” for the subsequent quantitative analysis.

Table 1: Fan laws [1]

Constants	Variable	Fan Laws	
Diameter (*D*) Density	Speed (*N*)	$G_2 = G_1 (N_2/N_1)$ $P_2 = P_1 (N_2/N_1)^2$	(FL1) (FL2)
Speed (*N*) Density	Diameter (*D*)	$G_2 = G_1 (D_2/D_1)^3$ $P_2 = P_1 (D_2/D_1)^2$	(FL3) (FL4)

Data

Catalogue work is well suited for student projects. Fortunately, my students did not have to take the (possibly rounded) catalogue values, but were able to evaluate a comprehensive table with experimental laboratory data from a German fan manufacturer. The result was a list of diameter, maximum static pressure (upper end of the vertical axis of the characteristic curve), maximum volume flow (right end of the horizontal axis) and speed for approx. 100 DC axial fans. These data were the basis for the following evaluation.

Table 2 shows the beginning of the list of my fan data. We know the diameter *D* of the fans, the maximum pressure *P*, the maximum volume flow and *G* and the corresponding nominal speed *N*.

Table 2: List with fan data

Name	*D* (mm)	*G* (liter/min)	*P* (Pa)	*N* (rpm)
Fan1	25	35.8	18.6	6000
Fan2	25	53.6	24.7	9000
Fan3	40	102.2	18.2	4300
Fan4	40	133.8	26.0	5400
…	…	…	…	…

“Diameter" in the data is the edge length of the fan housing. For the same diameter there are fans with different nominal speeds. For illustration Fig. 1 shows the raw data in *D*-*N* and *G*-*P* diagrams.

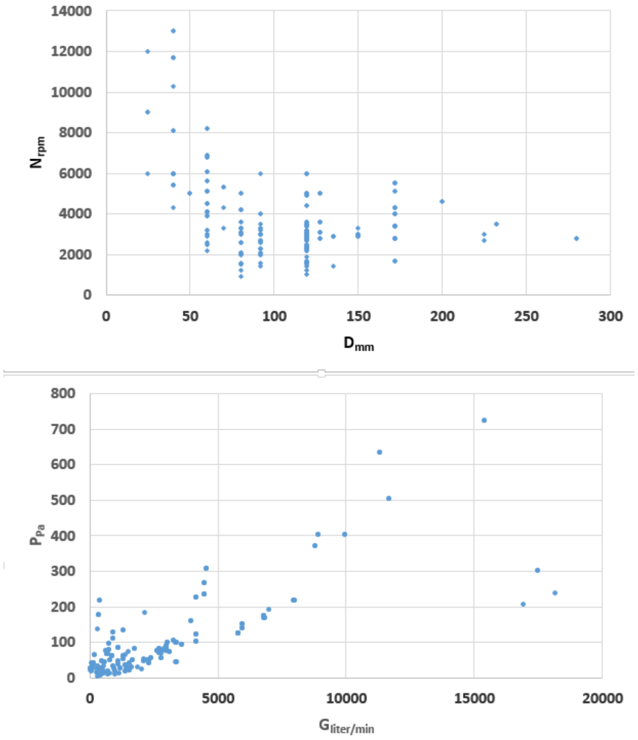

Figure 1. Fan raw data for this study.

Now we use the fan laws to get order into the point cloud.

Volume flow

The volume flow *G* should scale with D^3 (FL3). In order to compare the volume flow of fans rotating at different speeds, we define a normalized quantity *G** according to fan law (FL1)

$$G^* := \frac{G_{liter/min}}{N_{rpm}} \tag{1}$$

and plot *G** in a log-log diagram (Fig. 2) against the size of the fan *D* (in mm). *G**(*D*) indeed follows the power law (FL3) very well. We are also able to write down the quantitative relation we are looking for

$$G_{liter/min}(D) \approx 6 \cdot 10^{-7} \times N_{rpm} \times D_{mm}^3 \quad . \tag{2}$$

The interpretation of the r.h.s. could be as follows: *N* * *D* is something like the rotational speed of the blades and D^2 is a channel cross-section. Velocity * area results in a volume flow. Thus $G/(N*D^3)$ has no dimension.

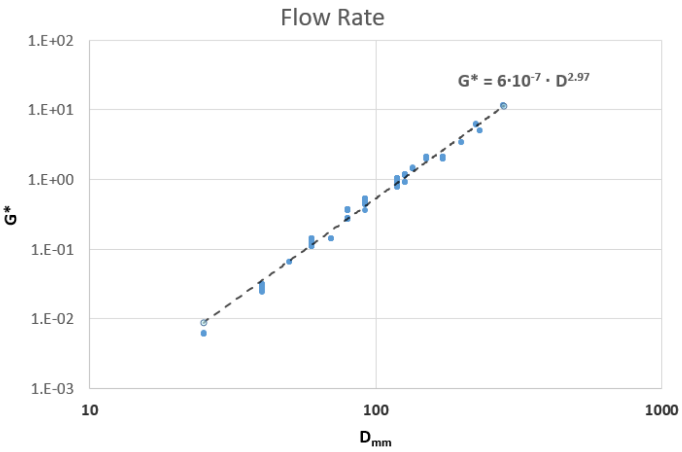

Figure 2: Normalized volume flow rate *G** vs. fan size *D.*

Static pressure

The static pressure should scale with D^2 (FL4). To restore comparability of fans rotating at different speeds, we define a normalized value *P** according to fan law (FL2)

$$P^* := \frac{P_{Pa}}{N_{rpm}^2} \tag{3}$$

and plot again a log-log diagram (Fig. 3). Now the correlation is not quite as perfect, but the D^2 law is clearly recognizable. Quantitatively we get

$$P_{Pa}(D) \approx 9 \cdot 10^{-10} \times N_{rpm}^2 \times D_{mm}^2 \quad . \tag{4}$$

The interpretation of the r.h.s. could be as follows: *N * D* is a velocity, velocity^2 * density is kinetic energy per volume is pressure (cf. Bernoulli equation $\rho v^2/2+P=\text{const}$).

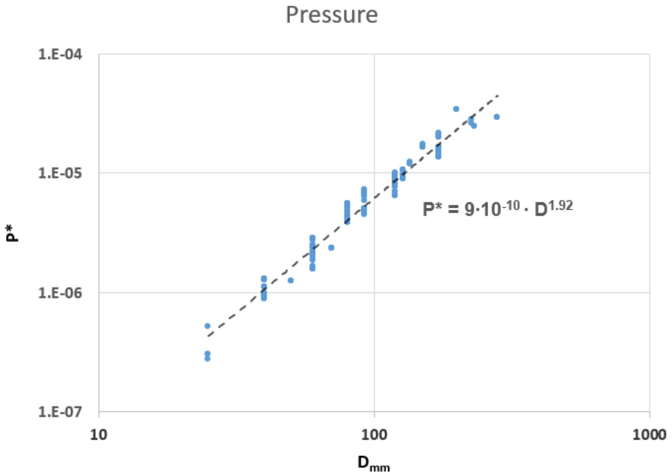

Figure 3: Normalized maximum static pressure *P** vs. fan size *D.*

Possible applications

What can you do with it now?

- You can enter fans with other motor technologies or blade geometries from this manufacturer in the diagrams.

- You can critically compare the fan correlation of other manufacturers with that of that manufacturer.
- You can identify typos in a catalog by displaying a fan as an outlier in the diagram.
- You can put marketing statements about new revolutionary fans into perspective.
- You can quickly estimate what type of fan you need for a particular volume flow or pressure.
- And best of all: parameterize quiet fans. The noise level is at least proportional N^5, i.e. the secret of quiet fans is slow fans. In order to generate a practicable volume flow and pressure, the fan must be correspondingly large. For cooling design see [2] and [3] to help find a suitable set of fan data. I should now mention that the diagrams above say nothing about the operating point.

Eqs. (2) and (4) help us now to bring systematics into the lower diagram of Fig. 1. Which combination of *N* and *D* belongs to which *G* and *P* point? Some little algebra is sufficient to find *P*(*G*;*D*) and *P*(*G*;*N*) formulae, functions that supply lines with constant *N* or constant *D* in the *P*-*G* diagram. Extract *N* from equation (2) and insert this *N* in equation (4)

$$P(G;D) = \frac{k_P}{k_G^2}\frac{G^2}{D^{2b-a}}$$

Extract *D* from equation (2) and put it into equation (4)

$$P(G;N) = \frac{k_P}{k_G^{a/b}} G^{a/b} N^{2-a/b} \ .$$

With k_G=6e-7, k_P=9e-10, *a*=2.965 (instead of 3), *b*=1.918 (instead of 2) we plot in Fig. 4 curves with constant *N* and *D* into the original data using these two formulas. One immediately recognizes, for example, where the large and where the slow fans are located. It is also noticeable that the *D*=40 mm line (as well as others) is systematically to the right of the corresponding points. This is because the fit curves (2) and (4) apply on average to all fans and have not specifically considered only the 40 series.

Back to the fan noise. For example, if the air volume is prescribed but the fan size can be varied, move at required *G* to the right until the speed corresponds to the estimated noise level and read off the approximate fan size and speed. This is only one of the possible application examples.

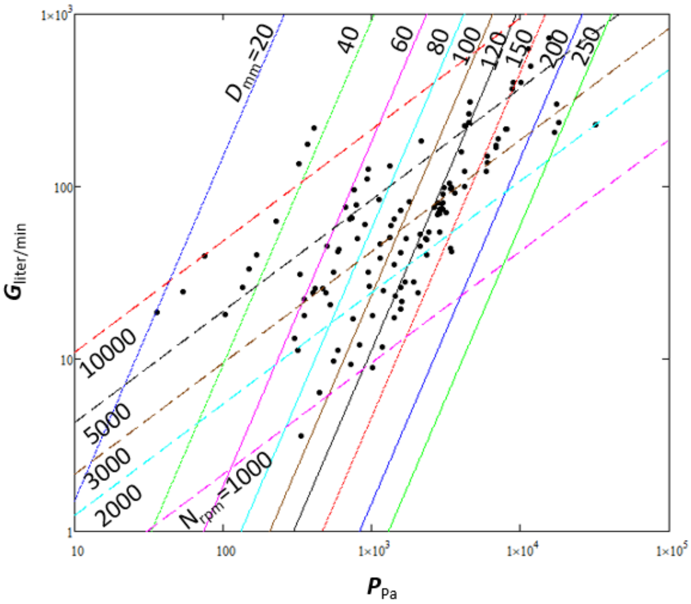

Figure 4: Searching for a suitable fan in a *P*-*G* graph. Points are the original fan data, solid lines are lines of constant fan size *D* and dashed lines are for constant speed *N*.

If you need to keep students busy with something, give them one of the points mentioned above as a task to extend the study to other vendors.

References.

[1] Turner, M.: https://www.electronics-cooling.com/1996/05/all-you-need-to-know-about-fans/ (1996)

[2] Ellison, G.N.: https://www.electronics-cooling.com/1995/10/fan-cooled-enclosure-analysis-using-a-first-order-method/ (1995)

[3] Ellison, G.N.: This appendix. Private communication (2018)

[3] Appendix: Corrections for [2]

Dr. Adam has graciously permitted me to provide some corrections to Ref. [2]. In that article the errors begin with the first equation on Page 19 and therefore lead to errors in the remaining airflow and well-mixed air temperature rises. The corrected equations follow:

$$\text{Power supply: } \dot{m}_{PS} = \dot{m}\sqrt{R_I / R_{PS}} = 0.0069\sqrt{3.60\times10^3 / 5.30\times10^5} = 5.69\times10^{-4}\, kg/s$$

$$\text{Single circuit board:} \dot{m}_{Brd} = \frac{1}{5}\left(\dot{m}-\dot{m}_{PS}\right) = \left(0.0069 - 5.69\times10^{-4}\right)/5 = 0.00127\, kg/s$$

$$\text{Circuit boards 1-4: } \Delta T = 9.65\times10^{-4} Q_{Brd} / \dot{m}_{Brd} = \left(9.65\times10^{-4}\right)\left(20W\right)/0.00127 = 15.2\ {}^{0}C$$

$$\text{Circuit board 5: } \Delta T = 9.65\times10^{-4} Q_{Brd} / \dot{m}_{Brd} = \left(9.65\times10^{-4}\right)\left(30W\right)/0.00127 = 22.8\ {}^{0}C$$

$$\text{Power supply:} \Delta T = 9.65\times10^{-4} Q_{PS} / \dot{m}_{PS} = \left(9.65\times10^{-4}\right)\left(25W\right)/5.69\times10^{-4} = 42.0\ {}^{0}C$$

About the Author

Johannes Adam is the owner of ADAM Research offering own software for the electro-thermal simulation of printed circuit boards as well as consulting through simulation. He received a diploma and doctorate in physics from University of Heidelberg, Germany. He has over 25 years of experience in thermal management fields and simulation software application.

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